Effects of nonlinear electrodynamics on neutral black holes

Speaker: Vorokhov A.V., postgraduate student,
e-mail: alexei.vorohov@yandex.ru;

Scientific supervisor: Sushkov S.V., Ph.D, professor, Head of the Department of Relativity;

Scientific advisor: Groshev D.E., Ph.D., associate professor;

Kazan Federal University, Institute of Physics
Department of Relativity

Научный семинар "АстроЧат"
Feb 20, 2026

❶ Introduction

❷ Fields around black holes

❸ Black holes in NED
       Origin
       Restrictions
       Properties

❹ Simulating black holes
       Classical cases
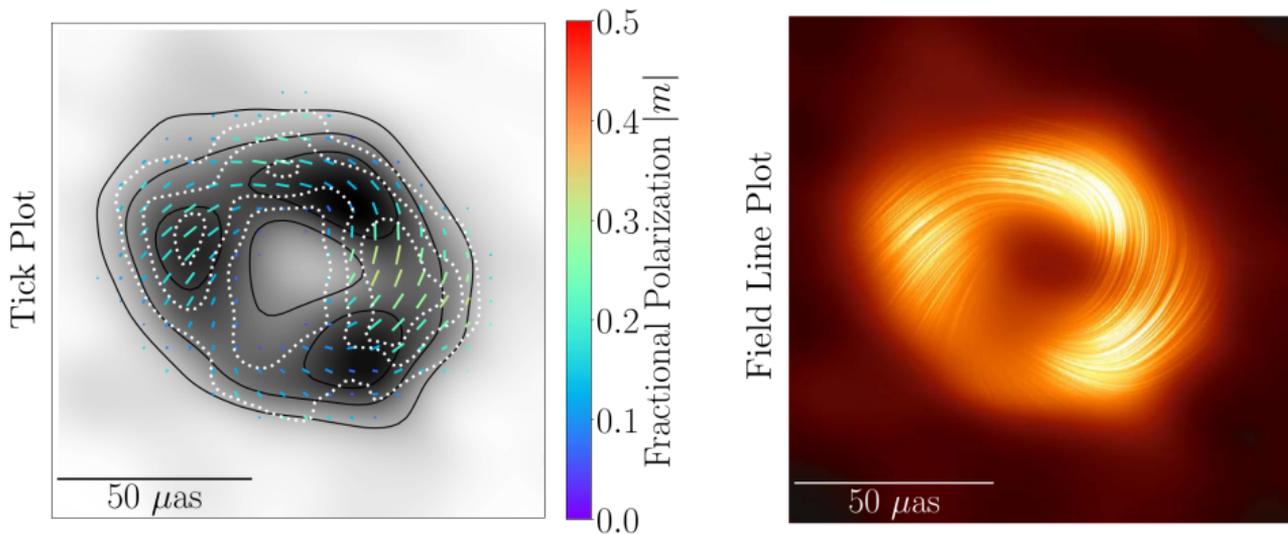   Exotic cases

❺ Results

Fig. 1: Polarimetric images of SgrA*. On the left panel: reconstructed linear polarization vectors and fractions, image intensity serves as background. On the right panel: image intensity is modulated by polarization (see[1] for details).

---

[1] The Event Horizon Telescope Collaboration et al. "First Sagittarius A* Event Horizon Telescope Results. VII. Polarization of the Ring". In: *The Astrophysical Journal Letters* 964.2 (Apr. 1, 2024), p. L25.

The idea that gravitational collapse can generate superstrong magnetic fields was proposed by Ginzburg in 1964 in the context of quasars[2].

- **Mechanism:** During the collapse of a highly conductive protostellar cloud, the magnetic flux is conserved ("frozen-in").

- **Amplification:** This leads to the amplification of the magnetic field (H) as the cloud's radius (R) shrinks:

$$H(R) \approx H_0 \left(\frac{R_0}{R} \right)^2 \tag{1}$$

- **Result:** For a cloud of $10^8 M_\odot$ collapsing to its gravitational radius (R_g), the initial field can be amplified to $\sim 10^5$ [T].

[2]Г. В.Л. "О магнитных полях коллапсирующих масс и природе сверхзвезд". In: 154.1 (1964), pp. 43–46.

In geometric units:

$$k = c = G = \hbar = 1, \quad \mu_0 = 4\pi \tag{2}$$

For linear (Maxwellian) electrodynamics, the Lagrangian density is expressed as:

$$\mathcal{L}_{\mathrm{ED}}(F, \tilde{F}) = \mathcal{L}_{\mathrm{Maxw}}(F) = -\frac{1}{16\pi} F \tag{3}$$

where F and $\tilde{F}$ are electromagnetic field invariants:

$$F = F^{\mu\nu} F_{\mu\nu} = 2(B^2 - E^2), \tag{4}$$

$$\tilde{F} = F^{\mu\nu} \tilde{F}_{\mu\nu} = 4E^\mu B_\mu \tag{5}$$

Evidently, this form is a linear function of F:

$$\partial_F \mathcal{L}_{\mathrm{ED}} = -\frac{1}{16\pi}. \tag{6}$$

In **nonlinear electrodynamics**, the Lagrangian density can, in general, take the form of an arbitrary function:

$$\mathcal{L}_{\mathrm{NED}}(F, \tilde{F}) = f(F, \tilde{F}) \tag{7}$$

with the common assumption that it reproduces Maxwell's theory in the weak-field limit:

$$\mathcal{L}_{\mathrm{NED}} \approx -\frac{1}{16\pi} F \quad \text{for } F, \tilde{F} \to 0 \tag{8}$$

Motivation for considering NED:

- More general algebraic symmetry;
- Extensions of Standard Model and search for new particles (e.g. axions);
- Dealing with singularities in classical electrodynamics (e.g., charge self-energy);
- Classical treatment of quantum corrections to electrodynamics;

For weak EM fields (with respect to the critical field), the Lagrangian can be represented by a power series - *parametric post-maxwellian extension* (*PPM*):

$$\mathcal{L}_{\text{PPM}} = -\frac{1}{16\pi}F + \frac{\xi}{32\pi}\left[\eta_1 F^2 + \eta_2 \tilde{F}^2\right] + \mathcal{O}(\xi^2) \tag{9}$$

Here, η_1 and η_2 are the PPM parameters, and ξ describes the critical field:

$$\xi = 1/B_q{}^2, \; B_q = \frac{m_e^2 c^2}{e\hbar} = 4.41 \cdot 10^9 \, [\text{T}] \tag{10}$$

In QED low-energy limit[3]:

$$\mathcal{L}_{\text{NED}} = \frac{1}{16\pi} \left\{ -F + h \left(F^2 + \frac{7}{4}\tilde{F}^2 \right) \right\} \tag{11}$$
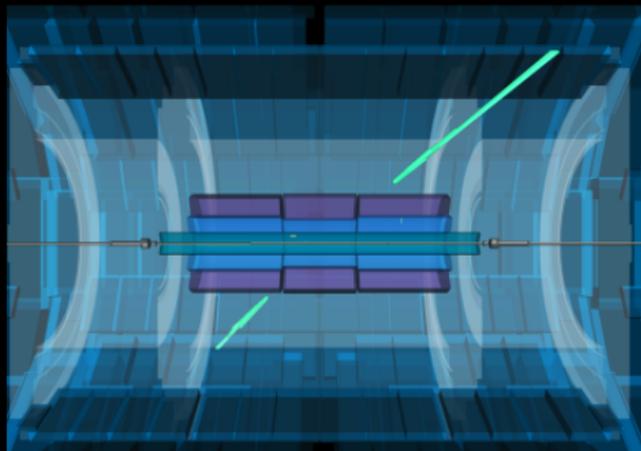
where:

$$h = \frac{2\hbar^3\alpha^2}{45\mu_0 m_e^4 c^5} = 1.32 \cdot 10^{-24}\,[\text{T}^{-2}] \tag{12}$$

[3] W. Heisenberg and H. Euler. "Folgerungen aus der Diracschen Theorie des Positrons". In:
Zeitschrift für Physik 98 (1936), pp. 714–732.

Fig. 2: ATLAS experiment scheme[4]. As shown in[5], Euler-Heisenberg electrodynamics agrees with the results of this experiment at the level of $\sim 4.2\sigma$.

[4]ATLAS Collaboration. "Evidence for light-by-light scattering in heavy-ion collisions with the ATLAS detector at the LHC". In: *Nature Physics* 13.9 (Sept. 2017), pp. 852–858.
[5]P. N. Akmansoy and L. G. Medeiros. "Constraining nonlinear corrections to Maxwell electrodynamics using gamma-gamma scattering". In: *Physical Review D* 99.11 (June 6, 2019),
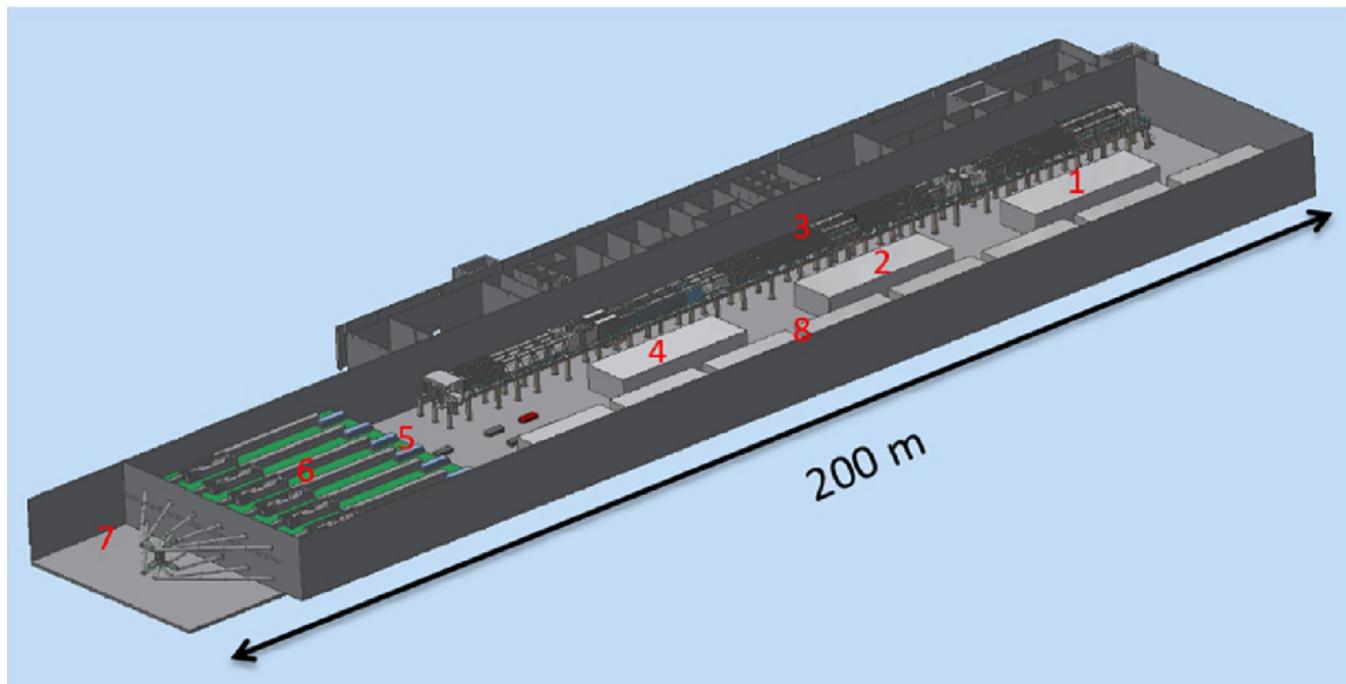
Fig. 3: The Exawatt laser facility, planned by the XCELS project[6].

---

[6]E. Khazanov et al. "eXawatt Center for Extreme Light Studies". In: *High Power Laser Science and Engineering* 11 (e73 2023).

- NED can be the source of regular black holes[7];
- NED changes GW spectra of NS and BH mergers[8];
- NED leads to polarization rotation and mode delay[9];

---

[7] E. Ayón-Beato and A. García. "Regular Black Hole in General Relativity Coupled to Nonlinear Electrodynamics". In: *Physical Review Letters* 80.23 (June 8, 1998), pp. 5056–5059, K. A. Bronnikov. "Regular black holes sourced by nonlinear electrodynamics". In: *Regular Black Holes: Towards a New Paradigm of the Gravitational Collapse*. Springer Singapore, 2023.

[8] D. P. Sorokin. "Introductory Notes on Non-linear Electrodynamics and its Applications". In: *Fortschritte der Physik* 70.7 (Aug. 2022), p. 2200092, M. Okyay and A. Övgün. "Nonlinear electrodynamics effects on the black hole shadow, deflection angle, quasinormal modes and greybody factors". In: *Journal of Cosmology and Astroparticle Physics* 2022.1 (Jan. 1, 2022), p. 009.

[9] V. I. Denisov, V. A. Sokolov, and S. I. Svertilov. "Vacuum non-linear electrodynamic polarization effects in hard emission of pulsars and magnetars". In: *Journal of Cosmology and Astroparticle Physics* 2017.9 (Sept. 4, 2017), pp. 004–004.

- Effective geometry for photons[10];
- Modifies GRMHD equations and thus affects accretion[11];
- NED is a common companion of various hypothetical paricles (e.g. axions), which are relevant in context of dark matter and early universe;

---

[10]M. Novello et al. "Geometrical aspects of light propagation in nonlinear electrodynamics". In: *Physical Review D* 61.4 (Jan. 13, 2000), p. 045001.

[11]Y. Kurmanov et al. "Accretion disks properties around regular black hole solutions obtained from non-linear electrodynamics". In: *Physics of the Dark Universe* 46 (Dec. 2024), p. 101566, H. Rehman et al. "Matter accretion onto the magnetically charged Euler–Heisenberg black hole with scalar hair". In: *The European Physical Journal C* 83.9 (Sept. 25, 2023), p. 856, H. Rehman et al. "Circular orbits of accretion flow around charged black hole coupled with a nonlinear electrodynamics field". In: *The European Physical Journal C* 84.9 (Sept. 28, 2024), p. 988.

- Study the possibility of constraining NED models using the observations of compact objects

Consider a spherically-symmetric metric:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{13}$$

Substituted into the Einstein equations, this leads to

$$f(r) = 1 - \frac{2m}{r} - \frac{8\pi G}{r}\int r^2 T_0^0(r)dr \tag{14}$$

Which for classical ED results in the Reissner-Nordstrom solution:

$$f(r) = 1 - \frac{2m}{r} + \frac{q^2}{r^2} \tag{15}$$

In this solution, the **electromagnetic field energy** effectively provides attracts geodesic away from center, resulting in smaller horizon, ISCO and photon sphere radii.

- The Penrose process can lead to the evaporation of a BH's charge, both electric and magnetic[12];

- On the other hand, an inverse process has been demonstrated - the accelerated motion of a BH in a galactic magnetic field can lead to the BH charging up[13];

- The EM fields of a BH's environment can often be described in terms of an effective n-pole charge of the BH;

---

[12]Z. Stuchlík, M. Kološ, and A. Tursunov. "Penrose Process: Its Variants and Astrophysical Applications". In: *Universe* 7.11 (Oct. 31, 2021), p. 416.

[13]P. Adari, R. Berens, and J. Levin. *Charging up Boosted Black Holes*. Dec. 4, 2022.
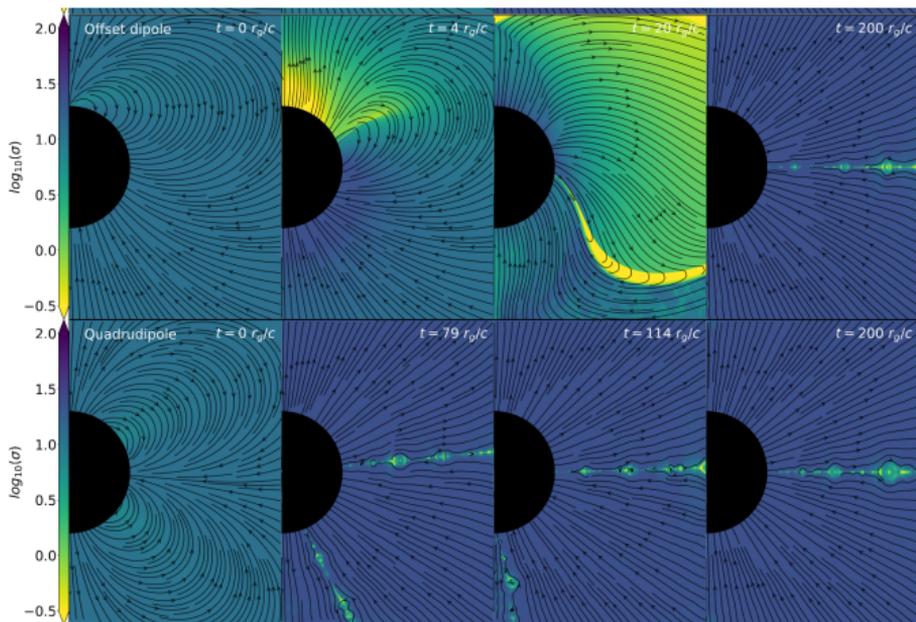
Fig. 4: A split-monopole magnetic field seems to be the most realistic EM field configuration for a BH with surrounding plasma, as was investigated in the works[14]. In a recent article[15], the universality of such a configuration is shown.

---

[14]R. M. Wald. "Black hole in a uniform magnetic field". In: *Physical Review D* 10.6 (Sept. 15, 1974), pp. 1680–1685, S. S. Komissarov. "Electrodynamics of black hole magnetospheres". In: *Monthly Notices of the Royal Astronomical Society* 350.2 (May 11, 2004), pp. 427–448.

[15]S. Selvi et al. *On the universality of the split monopole black hole magnetosphere*. Apr. 14, 2025.

Bardeen's regular BH solution[16]:

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2 \tag{16}$$

$$f(r) = 1 - \frac{2mr^2}{(r^2 + q^2)^{3/2}} + \frac{q^2 r^2}{(r^2 + q^2)^2} \tag{17}$$

This solution was first physically justified by NED[17], where the parameter q was identified as the dimensionless charge of the black hole.

[16] J. Bardeen. "Non-singular general relativistic gravitational collapse". In: *Proceedings of the 5th International Conference on Gravitation and the Theory of Relativity*. 5th International Conference on Gravitation and the Theory of Relativity. Tiflis: Tbilisi : Pub. House of Tbilisi University, 1968, p. 87.

[17] E. Ayón-Beato and A. García. "Regular Black Hole in General Relativity Coupled to Nonlinear Electrodynamics". In: *Physical Review Letters* 80.23 (June 8, 1998), pp. 5056–5059.

Causality: *Elementary excitations of the EM field must have a group velocity $\leq c$;*

Unitarity: *The propagator's residue must be positive.*

These conditions lead to the following restrictions[18]:

$$1 - \partial_F \mathcal{L} + 2F \partial_{\tilde{F}\tilde{F}} \mathcal{L} \geq 0, \tag{18}$$

$$1 - \partial_F \mathcal{L} \geq 0, \quad \partial_{\tilde{F}\tilde{F}} \mathcal{L} \geq 0, \tag{19}$$

$$1 - \partial_F \mathcal{L} - 2F \partial_{FF} \mathcal{L} \geq 0, \quad \partial_{FF} \mathcal{L} \geq 0. \tag{20}$$

---

[18] A. E. Shabad and V. V. Usov. "Effective Lagrangian in nonlinear electrodynamics and its properties of causality and unitarity". In: *Physical Review D* 83.10 (May 5, 2011), p. 105006, eq. (27-31).

The classical set of energy conditions[19]:

WEC: *local mass-energy density is non-negative*

$$T_{\mu\nu}U^\mu U^\nu \geq 0 \quad \forall \text{ timelike } U^\mu \tag{21}$$

DEC: *energy travels at speeds not greater than c*

$$T_{\mu\nu}U^\mu U^\nu \geq 0, \; V^\mu V_\mu \leq 0 \quad \forall \text{ timelike } U^\mu \tag{22}$$

here $V^\mu = -T^\mu_{.\nu}U^\nu$ - 4-vector of energy flux

SEC: *gravity always attracts*

$$\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right)U^\mu U^\nu \geq 0 \quad \forall \text{ timelike } U^\mu \tag{23}$$

[19]P. Martin-Moruno and M. Visser. "Classical and semi-classical energy conditions". In: *Wormholes, Warp Drives and Energy Conditions*. Springer International Publishing, Mar. 29, 2017, pp. 193–214.

The most systematic study of BH properties in NED[20]:

1. No-go theorems for charged BHs in $\mathcal{L}(F, \tilde{F})$ and $\mathcal{L}(F, J_4)$[21] NED theories;

2. Regular BHs with electric charge $q_e \neq 0$ are possible only in theories that have no Maxwellian limit;

3. Such solutions are possible for BHs with magnetic charge $q_m \neq 0$;

4. In the vicinity of the regular center, causality and unitarity conditions are inevitably violated.

---

[20] K. A. Bronnikov. "Regular black holes sourced by nonlinear electrodynamics". In: *Regular Black Holes: Towards a New Paradigm of the Gravitational Collapse.* Springer Singapore, 2023.

[21] $J_4 = F^{\mu\nu} F_{\nu\alpha} F^{\alpha\beta} F_{\beta\mu}$

Stress-energy tensor:

$$T^{\mu\nu} = T^{\mu\nu}_{\text{EM}} = -\mathcal{L}g^{\mu\nu} - 4\mathcal{L}_F F^{\mu\xi}F^\nu_\xi - 4\mathcal{L}_{\tilde{F}} F^{\mu\xi}F^{*\nu}_\xi \tag{24}$$

In case of spherical symmetry:

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2, \tag{25}$$

we obtain corrections to the classical Reissner-Nordstrom solution:

$$f(r) = 1 - \frac{2m}{r} - \frac{8\pi G}{r}\int r^2 T^0_0(r)dr \tag{26}$$

Effective geometry: photon-photon scattering can be described in purely geometrical terms[22].

For theories of type $\mathcal{L} = \mathcal{L}(F)$:

$$g_{\text{eff}}^{\mu\nu} = \mathcal{L}_{,F} g^{\mu\nu} - 4\mathcal{L}_{,FF} F^{\mu\alpha} F_{\alpha}^{\nu} \tag{27}$$

In theories with $\mathcal{L} = \mathcal{L}(F, \tilde{F})$, when $\tilde{F} \neq 0$, the metric depends on photon helicity: two different metrics $g_{+}^{\mu\nu}$ and $g_{-}^{\mu\nu}$ arise.

---

[22]M. Novello et al. "Geometrical aspects of light propagation in nonlinear electrodynamics". In: *Physical Review D* 61.4 (Jan. 13, 2000), p. 045001.

Vorokhov A.V.

Two types of tasks[23]:

- Physics of the object and its environment (*what happens?*);
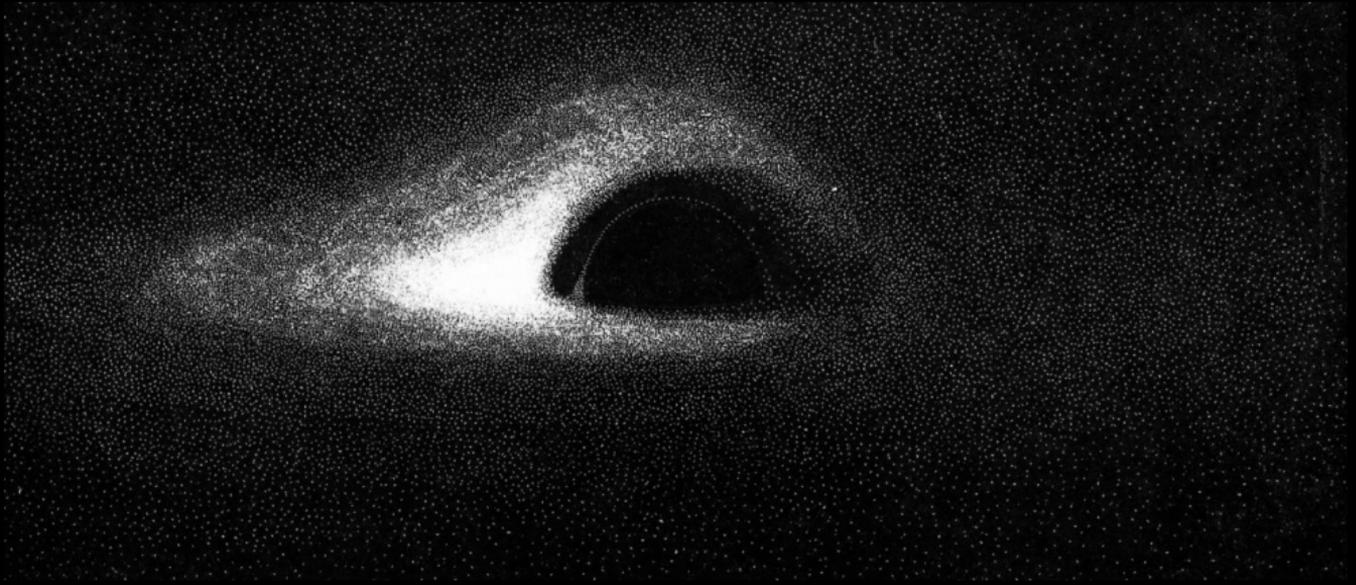- Modeling of observations (*what will we see?*)

[23]C. Fendt. "Approaching the Black Hole by Numerical Simulations". In: *Universe* 5 (2019).

Fig. 5: Image of a BH with a thin accretion disk by J.P. Luminet[24]. It was drawn manually by the author based on computer modeling.

---

[24] J.-P. Luminet. "Image of a Spherical Black Hole with Thin Accretion Disk". In: *Astronomy & Astrophysics* 75 (1979), pp. 228–235.
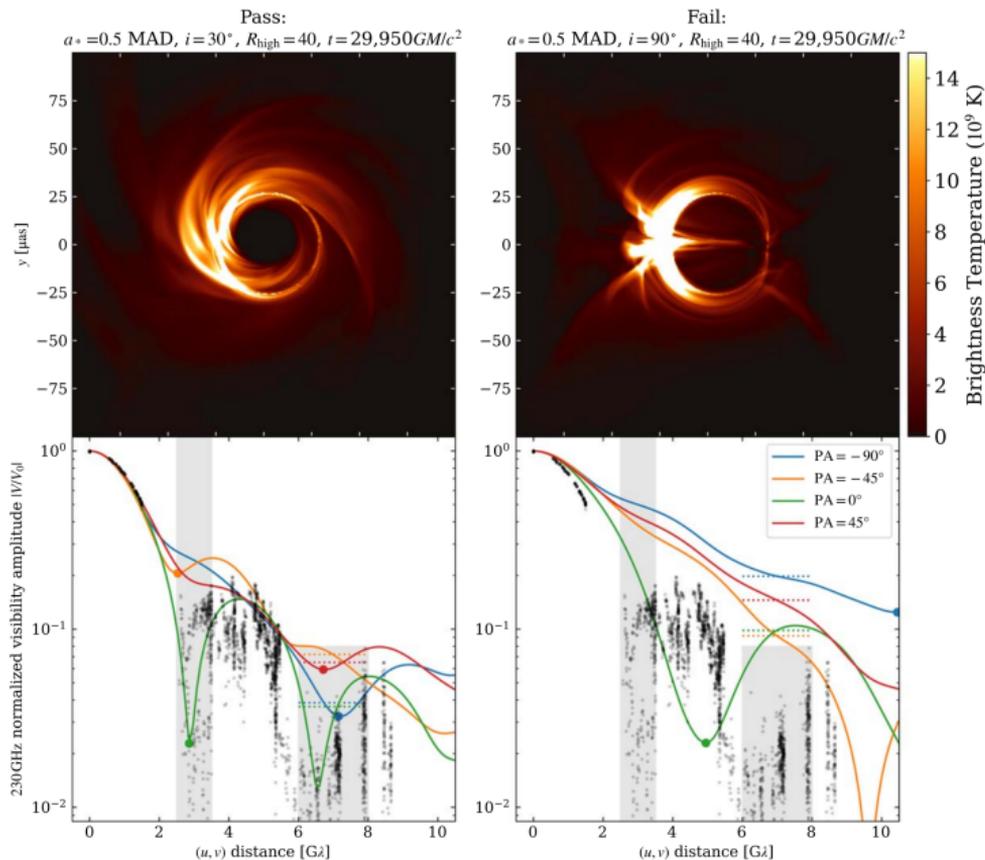
Fig. 6: GRRT images of a black hole and their interferometric signatures from the article [26]. A MAD accretion model was used;

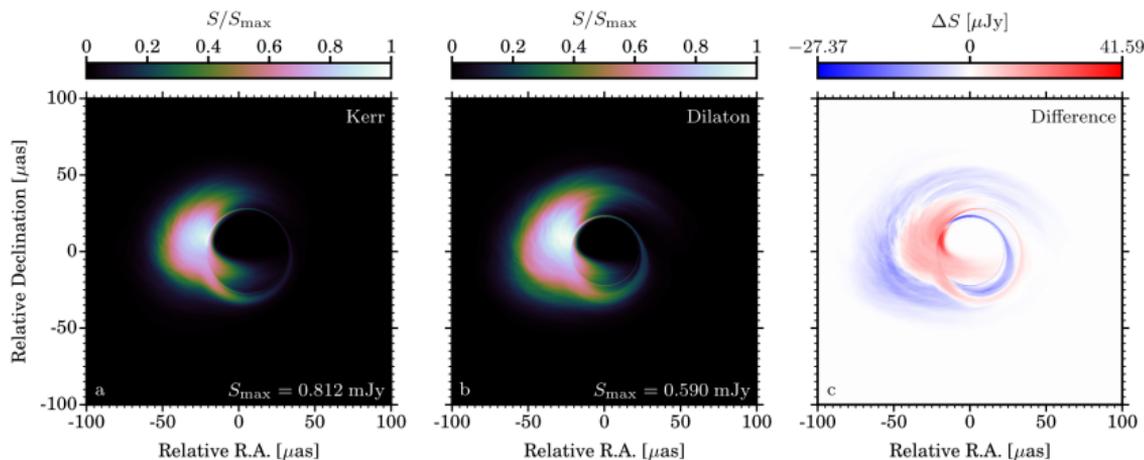Today, the term **exotic compact object** for alternatives to classical BH and NS solutions become established[25].

Fig. 7: Time-averaged images of a Kerr BH ($a = 0.6$) and a non-rotating BH in axion-dilaton gravity ($a \to 0, b \sim 0.5$) according to the article[26].

[25]C. Bambi et al., eds. *New Frontiers in GRMHD Simulations*. Springer Series in Astrophysics and Cosmology. Singapore: Springer Nature Singapore, 2025, ch. 18.

[26]Y. Mizuno et al. "The Current Ability to Test Theories of Gravity with Black Hole Shadows". In: *Nature Astronomy* 2.7 (Apr. 16, 2018), pp. 585–590.
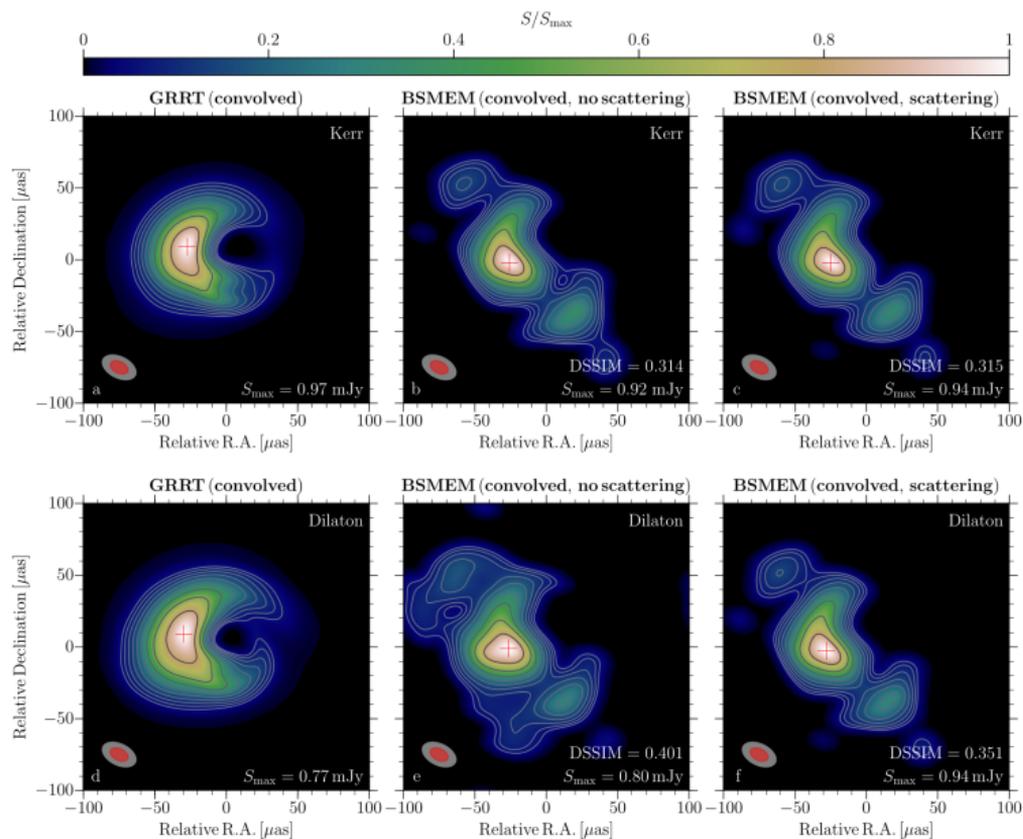
Fig. 8: Images from Fig. 7 after considering: (a, d) limited VLBI resolution; (b, e) limited (u, v)-coverage; and (c, f) interstellar light scattering.
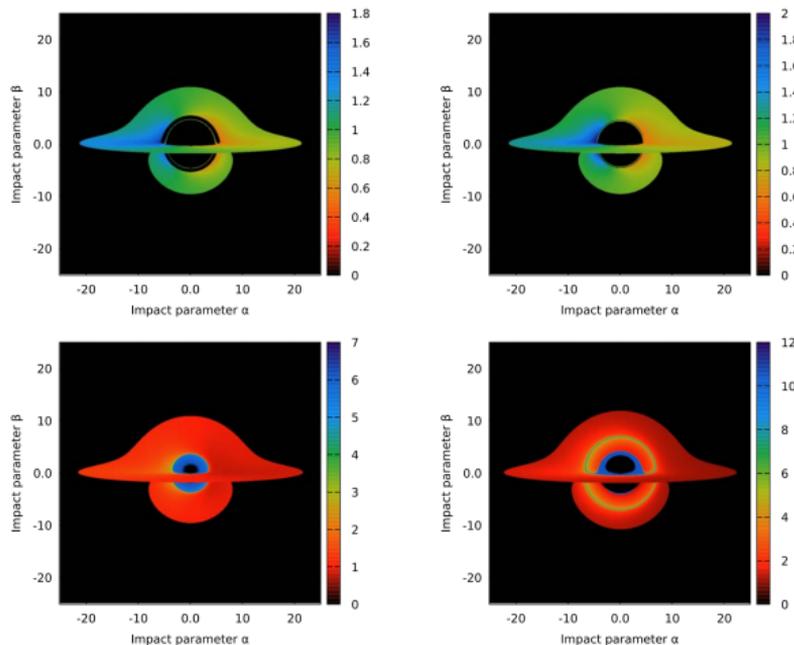
Fig. 9: Images of a thin Keplerian disk for different regular BH models from the article[27]. The parameters α and β are in units of M, and the color gradient represents redshift. The observation angle is θ_o. The magnetic charge values (q_m/M) are (from left to right, top to bottom): $0.2, 0.3, 0.4, 1.0$.

[27]Z. Stuchlík, J. Schee, and D. Ovchinnikov. "Generic Regular Black Holes Related to Nonlinear Electrodynamics with Maxwellian Weak-field Limit: Shadows and Images of Keplerian Disks". In: *The Astrophysical Journal* 887.2 (Dec. 20, 2019), p. 145.
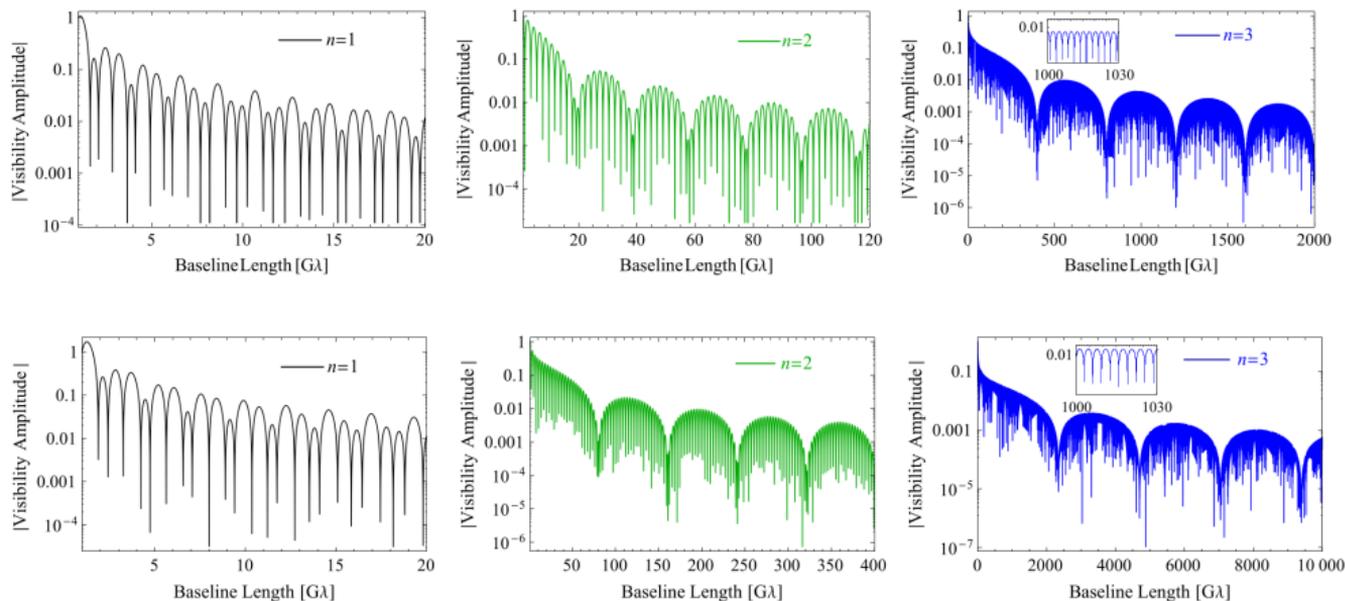
Fig. 10: Interferometric signatures of photon rings (up to the 3rd order) for a regular Bardeen BH and for an ultracompact object, with consideration of the effective geometry[28].

[28] R. K. Walia. "Exploring Nonlinear Electrodynamics Theories: Shadows of Regular Black Holes and Horizonless Ultra-Compact Objects". In: *Physical Review D* 110.6 (Sept. 18, 2024), p. 064058.

A. Vorokhov and D. Groshev. "Black hole image in Heisenberg-Euler electrodynamics". In: *Space, Time and Fundamental interactions* 1 (2024), pp. 132–133
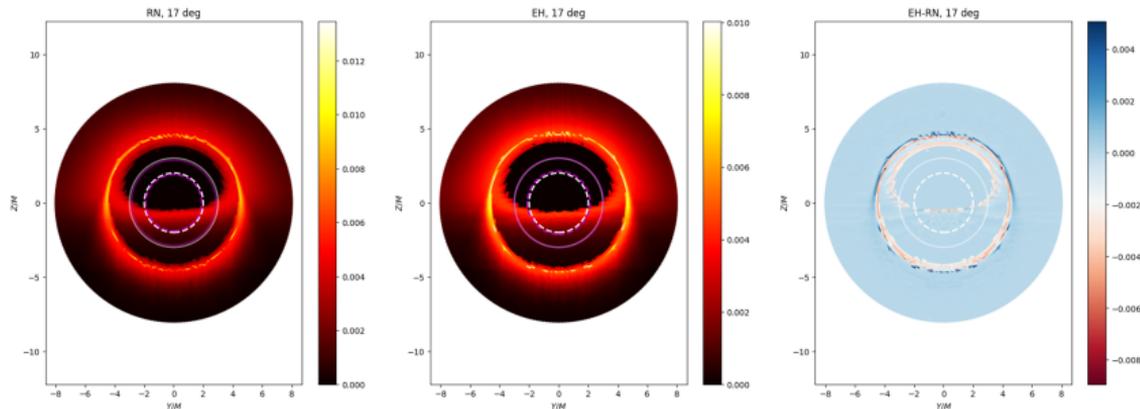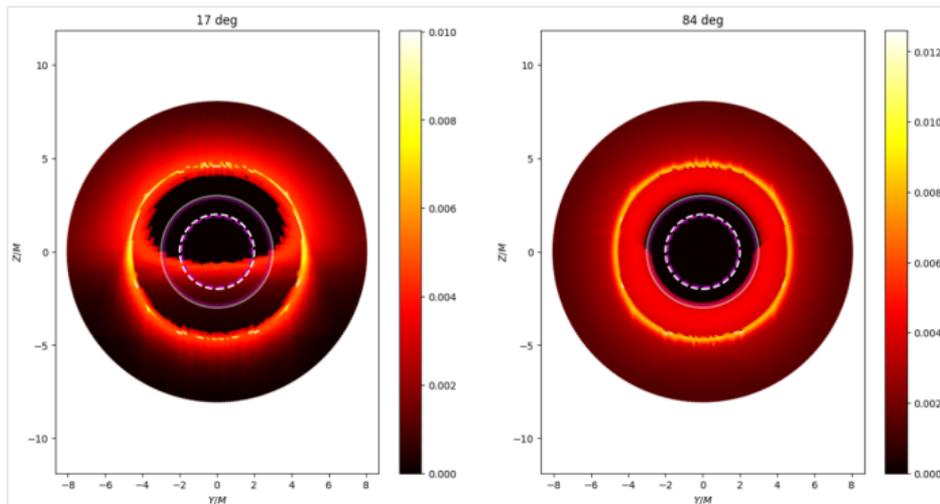

Fig. 11: Images of Reissner-Nordstrom BH in classical (left panel) and Euler-Heisenberg (central panel) electrodynamics. Charges are equal, $Q = 0.45M$. Image difference is presented on the right panel.

A. V. Vorokhov. *BHTRACE - a tool for GRRT in effective geometry, sourced by nonlinear electrodynamics.* 2025 https://github.com/alexeivorohov/bhtrace

GitHub - alexeivorohov/bhtrace: Relativistic objects imaging, powered by PyTorch
github.com/alexeivor...

Motivation:

- Create *pythonic* GRRT tool as simple and extensible as NumPy;
- Support of any kinds of custom metrics;
- Reliable baseline for study and comparison for all modified models of black holes;

- Uses PyTorch for robust tensor operations and easy itegration with machine learning techniques;
- Backward-differentiable ray-tracing and grrt(*);
- Hardware acceleration (cuda, rocm, mps, ...);
- Clean class hierarchy, easy extension of any base classes;
- NumPy-like API (*);
- Follows NumPy/SciPy documentation style (*);
- Configuration system and cli tool for easy operation on remote machines (*);

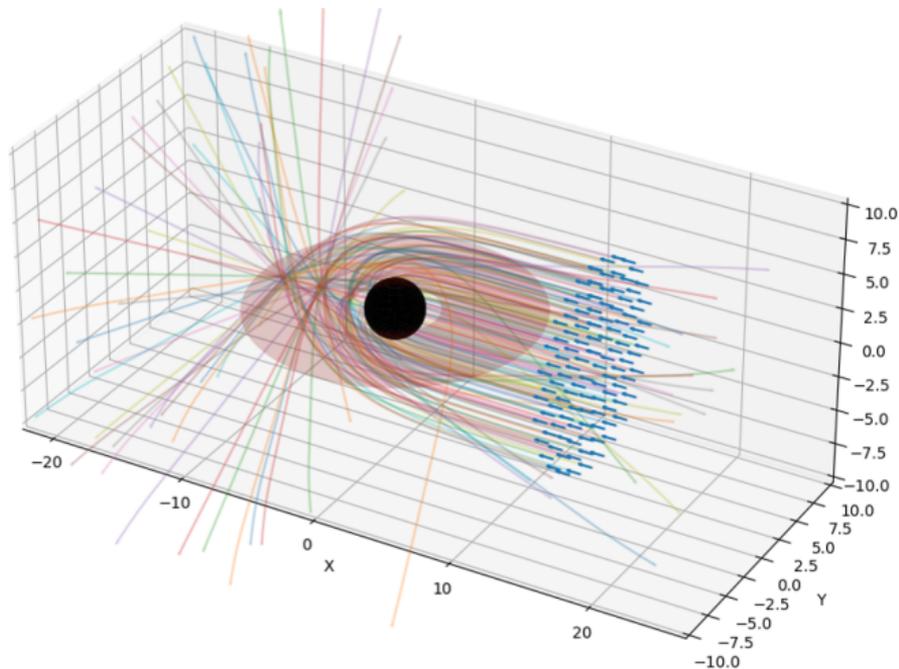Fig. 12: Example of ray-traced photon trajectories. The initial conditions are set on the observer's image plane ($X = 16$ for this example), and the photon geodesics are then integrated backward in time to determine photon trajectories near the compact object.
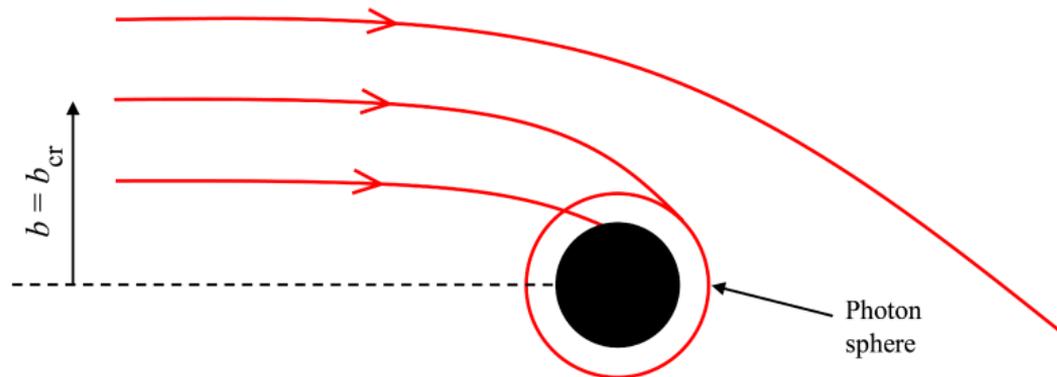
Fig. 13: Lensing function illustates the dependence of photon deflection by gravitational field from impact parameter. For most of black hole models lensing function tends to $+\infty$ as $b \to b_{cr}$ - such impact factors correspond to quasi-stable photon orbits (photon sphere) of the black hole.
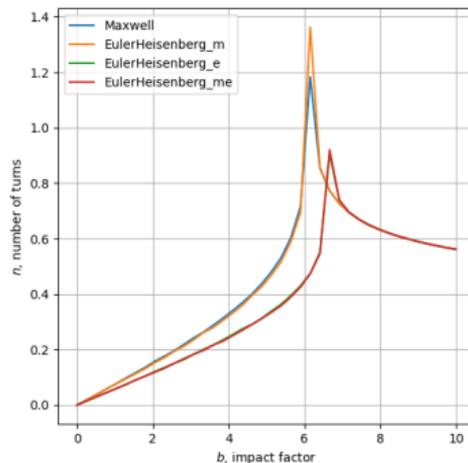
Fig. 14: Lensing function for charged BH in Maxwellian electrodynamics (blue) and in Euler-Heisenberg electrodynamics in different setups (orange - only mass contribution, green - only effective geometry contribution, red - both contributions)[29]. Corrections from effective geometry outperform mass corrections.

---

[29]A. Vorokhov and D. Groshev. "Light transport and images of compact objects in effective geometry, sourced by NED". In: *Международная конференция по гравитации, космологии и астрофизике «RusGrav-18». Тезисы докладов.* 18th RUSSIAN GRAVITATIONAL CONFERENCE International Conference on Gravitation, Astrophysics and Cosmology. Kazan: KPFU, 2024, pp. 46–47.

Lensing function in effective geometry

Fig. 15: Two-side lensing function of photons, moving in $z = 0$ plane in Kerr spacetime ($a = 0.6$) with (`eh_eff`) and without (`base`) effective geometry, sourced by stong split-monopole magnetic field in Euler-Heisenberg electrodynamics.
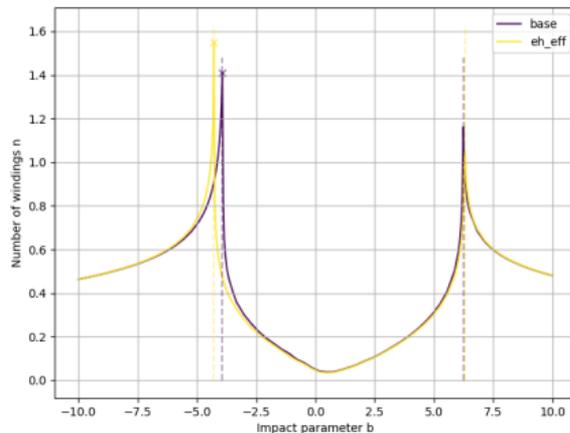
Physically consistent matter simulations in `bhtrace` are not yet stable. Nevertheless, qualitative features of the effective geometry can already be illustrated using a simple "test" medium — a static spherical gas shell with uniform density and temperature.
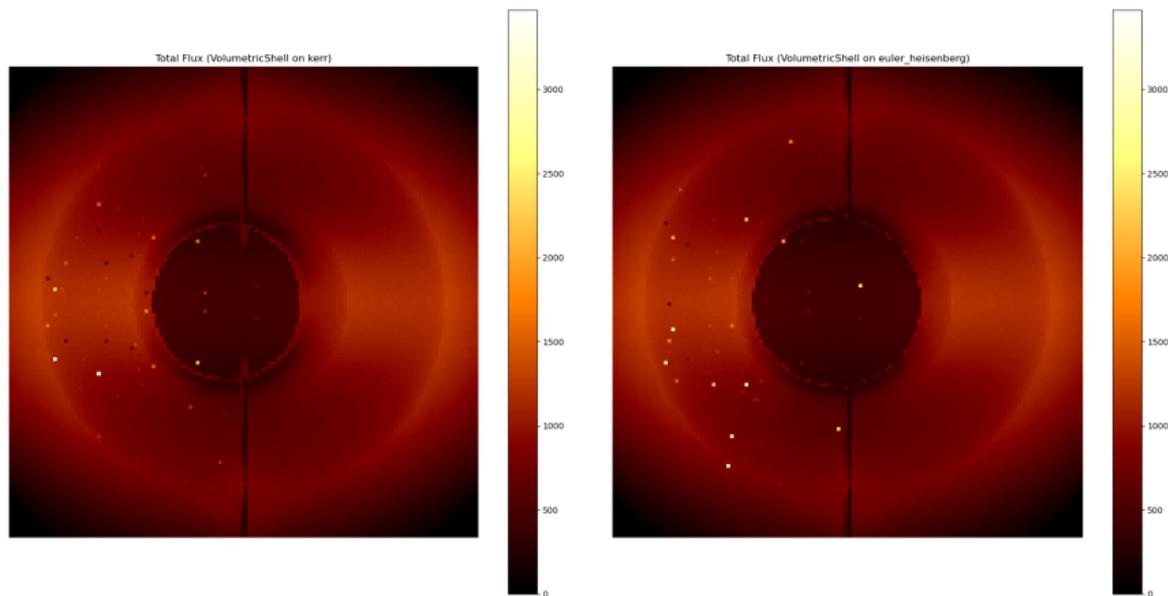
Fig. 16: Bolometric images of a Kerr black hole with spin parameter $a = 0.6$ (moderately rotating) foro observation angle $\theta = \pi/2$. Left panel: purely classical case; right panel: strong split-monopole magnetic field affects photon motion through the effective geometry, effectively increasing size of BH shadow. Noticeable numerical artifacts appear along the polar axis (typical for spherical-coordinate GRRT schemes); mitigating these effects and improving stability are ongoing tasks.

[1] The Event Horizon Telescope Collaboration et al. "First Sagittarius A* Event Horizon Telescope Results. VII. Polarization of the Ring". In: *The Astrophysical Journal Letters* 964.2 (Apr. 1, 2024), p. L25.

[2] Г. В.Л. "О магнитных полях коллапсирующих масс и природе сверхзвезд". In: 154.1 (1964), pp. 43–46.

[3] W. Heisenberg and H. Euler. "Folgerungen aus der Diracschen Theorie des Positrons". In: *Zeitschrift für Physik* 98 (1936), pp. 714–732.

[4] ATLAS Collaboration. "Evidence for light-by-light scattering in heavy-ion collisions with the ATLAS detector at the LHC". In: *Nature Physics* 13.9 (Sept. 2017), pp. 852–858.

[5] P. N. Akmansoy and L. G. Medeiros. "Constraining nonlinear corrections to Maxwell electrodynamics using gamma-gamma scattering". In: *Physical Review D* 99.11 (June 6, 2019), p. 115005.

[6] E. Khazanov et al. "eXawatt Center for Extreme Light Studies". In: *High Power Laser Science and Engineering* 11 (e73 2023).

[7] E. Ayón-Beato and A. García. "Regular Black Hole in General Relativity Coupled to Nonlinear Electrodynamics". In: *Physical Review Letters* 80.23 (June 8, 1998), pp. 5056–5059.

[8] K. A. Bronnikov. "Regular black holes sourced by nonlinear electrodynamics". In: *Regular Black Holes: Towards a New Paradigm of the Gravitational Collapse.* Springer Singapore, 2023.

[9] D. P. Sorokin. "Introductory Notes on Non-linear Electrodynamics and its Applications". In: *Fortschritte der Physik* 70.7 (Aug. 2022), p. 2200092.

[10] M. Okyay and A. Övgün. "Nonlinear electrodynamics effects on the black hole shadow, deflection angle, quasinormal modes and greybody factors". In: *Journal of Cosmology and Astroparticle Physics* 2022.1 (Jan. 1, 2022), p. 009.

[11] V. I. Denisov, V. A. Sokolov, and S. I. Svertilov. "Vacuum non-linear electrodynamic polarization effects in hard emission of pulsars and magnetars". In: *Journal of Cosmology and Astroparticle Physics* 2017.9 (Sept. 4, 2017), pp. 004–004.

[12] M. Novello et al. "Geometrical aspects of light propagation in nonlinear electrodynamics". In: *Physical Review D* 61.4 (Jan. 13, 2000), p. 045001.

[13] Y. Kurmanov et al. "Accretion disks properties around regular black hole solutions obtained from non-linear electrodynamics". In: *Physics of the Dark Universe* 46 (Dec. 2024), p. 101566.

[14] H. Rehman et al. "Matter accretion onto the magnetically charged Euler–Heisenberg black hole with scalar hair". In: *The European Physical Journal C* 83.9 (Sept. 25, 2023), p. 856.

[15] H. Rehman et al. "Circular orbits of accretion flow around charged black hole coupled with a nonlinear electrodynamics field". In: *The European Physical Journal C* 84.9 (Sept. 28, 2024), p. 988.

[16] Z. Stuchlík, M. Kološ, and A. Tursunov. "Penrose Process: Its Variants and Astrophysical Applications". In: *Universe* 7.11 (Oct. 31, 2021), p. 416.

[17] P. Adari, R. Berens, and J. Levin. *Charging up Boosted Black Holes*. Dec. 4, 2022.

[18] R. M. Wald. "Black hole in a uniform magnetic field". In: *Physical Review D* 10.6 (Sept. 15, 1974), pp. 1680–1685.

[19] S. S. Komissarov. "Electrodynamics of black hole magnetospheres". In: *Monthly Notices of the Royal Astronomical Society* 350.2 (May 11, 2004), pp. 427–448.

[20] S. Selvi et al. *On the universality of the split monopole black hole magnetosphere.* Apr. 14, 2025.

[21] J. Bardeen. "Non-singular general relativistic gravitational collapse". In: *Proceedings of the 5th International Conference on Gravitation and the Theory of Relativity.* 5th International Conference on Gravitation and the Theory of Relativity. Tiflis: Tbilisi : Pub. House of Tbilisi University, 1968, p. 87.

[22] A. E. Shabad and V. V. Usov. "Effective Lagrangian in nonlinear electrodynamics and its properties of causality and unitarity". In: *Physical Review D* 83.10 (May 5, 2011), p. 105006.

[23] P. Martin-Moruno and M. Visser. "Classical and semi-classical energy conditions". In: *Wormholes, Warp Drives and Energy Conditions.* Springer International Publishing, Mar. 29, 2017, pp. 193–214.

[24] C. Fendt. "Approaching the Black Hole by Numerical Simulations". In: *Universe* 5 (2019).

[25] J.-P. Luminet. "Image of a Spherical Black Hole with Thin Accretion Disk". In: *Astronomy & Astrophysics* 75 (1979), pp. 228–235.

[26]   Event Horizon Telescope Collaboration et al. "First Sagittarius A* Event
       Horizon Telescope Results. V. Testing Astrophysical Models of the Galactic
       Center Black Hole". In: *The Astrophysical Journal Letters* 930.2 (May 1,
       2022), p. L16.

[27]   C. Bambi et al., eds. *New Frontiers in GRMHD Simulations*. Springer Series
       in Astrophysics and Cosmology. Singapore: Springer Nature Singapore, 2025.

[28]   Y. Mizuno et al. "The Current Ability to Test Theories of Gravity with Black
       Hole Shadows". In: *Nature Astronomy* 2.7 (Apr. 16, 2018), pp. 585–590.

[29]   Z. Stuchlík, J. Schee, and D. Ovchinnikov. "Generic Regular Black Holes
       Related to Nonlinear Electrodynamics with Maxwellian Weak-field Limit:
       Shadows and Images of Keplerian Disks". In: *The Astrophysical Journal* 887.2
       (Dec. 20, 2019), p. 145.

[30]   R. K. Walia. "Exploring Nonlinear Electrodynamics Theories: Shadows of
       Regular Black Holes and Horizonless Ultra-Compact Objects". In: *Physical
       Review D* 110.6 (Sept. 18, 2024), p. 064058.

[31]   A. Vorokhov and D. Groshev. "Black hole image in Heisenberg-Euler
       electrodynamics". In: *Space, Time and Fundamental interactions* 1 (2024),
       pp. 132–133.

[32] A. V. Vorokhov. *BHTRACE - a tool for GRRT in effective geometry, sourced by nonlinear electrodynamics.* 2025.

[33] A. Vorokhov and D. Groshev. "Light transport and images of compact objects in effective geometry, sourced by NED". In: *Международная конференция по гравитации, космологии и астрофизике «RusGrav-18». Тезисы докладов.* 18th RUSSIAN GRAVITATIONAL CONFERENCE International Conference on Gravitation, Astrophysics and Cosmology. Kazan: KPFU, 2024, pp. 46–47.