ISSN 0236-2457
ASTRONOMICHESKII TSIRKULYAR

Published by the Eurasian Astronomical Society
and Sternberg Astronomical Institute

No. 1627, 2015 October 19

Asymptotical Trajectories for Collinear Libration Points of the Sun–Mars System

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Received September 14, 2015

Abstract. Three first coefficients of the series for powers of an arbitrary constant, which represent four asymptotical solutions for any collinear libration point of the Sun–Mars system, have been obtained. Numerical integration was applied for deriving the asymptotical trajectory in points distant from the libration point.

In order to place a satellite in the vicinity of a libration point of the Sun–Mars system, it is necessary to study flight trajectories from Mars to the vicinity of the libration point. The trajectories in which the space vehicle reaches the libration point vicinity with zero relative velocity are of utmost interest. To study such trajectories in the vicinity of the libration points, one can use Liapounov [1] and Poincaré [2] researches on solving sets of differential equations that determine the orbits asymptotically leaving or reaching the equilibrium position. Here we have applied the model of the plane circular restricted three-body problem. Asymptotical solutions are considered in the form of a series in powers of an arbitrary constant obtained in [3]. Four asymptotical solutions exist for every libration point. Two solutions are asymptotically approaching the considered Lagrange solution at $t \to \infty$, and two others are receding. Taking the Sun–Mars mass ratio as 3082879.63, we obtain the following asymptotical solutions for the Sun–Mars system:

for L1: $x1 = 33.709539977990440\, mn \exp[-n\alpha_1(t-\tau)] +$
$113507.997855432500000 \exp[-2n\alpha_1(t-\tau)] + 597566101.2488642\, mn \exp[-3n\alpha_1(t-\tau)]$,
$x_2 = 18.114864786133790\, m \exp[-n\alpha_3(t-\tau)] -$
$598.612836290641300\, n \exp[-2n\alpha_3(t-\tau)] - 168796385.5639517\, m, \exp[-3n\alpha_3(t-\tau)]$,

for L2: $x_1 = 32.974143966522110\, mn \exp[-n\alpha_2(t-\tau)] -$
$109565.230514167900000 \exp[-2n\alpha_2(t-\tau)] + 563216878.790451\, mn \exp[-3n\alpha_2(t-\tau)]$,
$x_2 = 17.886575079163870m \exp[-n\alpha_3(t-\tau)] - 0.02063873735896047n \exp[-2n\alpha_3(t-\tau)] -$
$159811738.3836557m \exp[-3n\alpha_3(t-\tau)]$,

for L3: $x_1 = 0.01031970371138370\, mn \exp[-n\alpha_3(t-\tau)] +$
$18.000063456039400 \exp[-2n\alpha_3(t-\tau)] - 188369.624374282900000\, mn \exp[-3n\alpha_3(t-\tau)]$,
$x_2 = 6.000008874677264\, m \exp[-n\alpha_3(t-\tau)] + 592.199500828665700\, n \exp[-2n\alpha_3(t-\tau)] -$
$36506531.30094195\, m \exp[-3n\alpha_3(t-\tau)]$,

where $\alpha_1 = 2.519776333044089$, $\alpha_2 = 2.496888299650090$, $\alpha_3 = -0.002579920203868829$ and m, n are integers taking the values $+1$ or -1.

For $n = +1$ ($m = \pm 1$) these formulas determine the orbits approaching the libration points while for $n = -1$ ($m = \pm 1$) they determine the two orbits receding from the libration points. It should be noted that the orbits indicated above are situated symmetrically with respect to the x_1 axis, namely, the incoming orbit corresponding to $m = 1$ and $n = 1$ is symmetrical to the outgoing orbit that corresponds to $m = -1$ and $n = -1$, while the incoming orbit that corresponds to $m = -1$ and $n = 1$ is symmetrical to the outgoing orbit corresponding to $m = 1$ and $n = 1$. The character of the orbits is shown in Fig. 1. The angle of inclination of the incoming orbit that corresponds to $m = 1$ and $n = 1$ is:
$\theta = 28°15'10''$ for L1, $\theta = 28°28'38''$ for L2, $\theta = 89°54'05''$ for point L3.

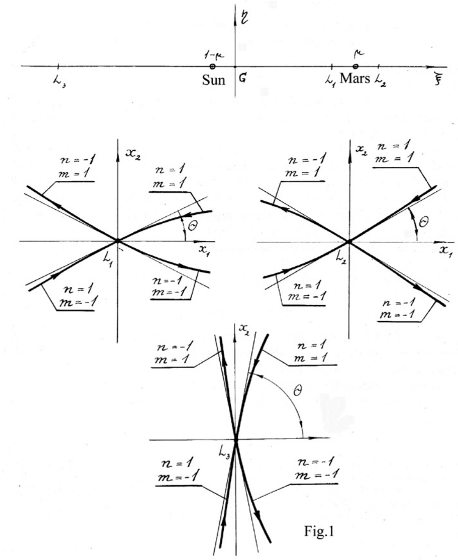

Fig.1

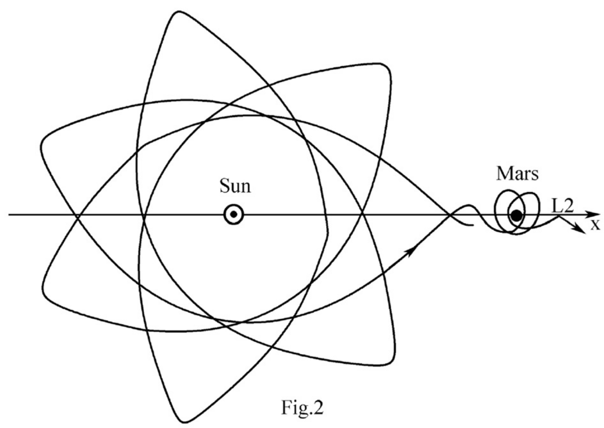

Fig.2

To obtain asymptotical trajectories for large distances from the libration points, numerical integration was applied. The character of two of the asymptotical trajectories is shown in Fig. 2. Other two trajectories are obtained by the symmetry rule. The units of measurement are chosen so that the sum of the finite masses, the gravitational constant, and the distance between the finite masses are taken as units. For astrodynamics problems the asymptotical trajectories in the I and IV quarters (see Fig. 1) apparently have no practical application. Two other asymptotical trajectories can be applied for a space station creation in the vicinity of the libration point.

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Асимптотические траектории для коллинеарных точек либрации системы Солнце–Марс

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Резюме. Получены три первых коэффициента разложения в ряд по степеням произвольной константы, которых представляют четыре асимптотических решения для любой коллинеарной точки либрации системы Солнце–Марс. Для определения асимптотической траектории в точках, удалённых от точки либрации, применено численное интегрирование.