ЭФФЕКТИВНОЕ ВРЕМЯ СТОХАСТИЗАЦИИ В СИСТЕМАХ БОЛЬШОГО ЧИСЛА ЗВЕЗД

Effective Stochastization Time for Stellar Systems with Large Number of Stars

Abstract. Для систем с большим числом звезд $N \gg 1$ использование выражения, предложенного Петровской, для закона распределения случайного ускорения в звездных системах приводит к следующему выражению для эффективного времени стохастизации: $\tau_e \propto \tau_c N^{1/3}/(\ln N)^{1/2}$, где τ_c – время пересечения.

Relaxation and stochastization in stellar systems were discussed recently by many authors (e. g. Kandrup H. E., 1990, ApJ, **364**, 420; Boccaletti D. *et al.*, A&A, 1999, **219**, 48; Merritt D., Ann. NY Acad. Sci., 2005, **1045**, 3; Rastorguev A. S. & Sementsov V. N., Astron. Lett., 2006, **32**, 16; Ossipkov L. P., Mess. St.Petersb. Univ., ser. 10, 2009, iss. 2, 93; Ovod D. V., Ast. Tsirk., 2012, No. 1571, 1). Concepts and methods of the ergodic theory were applied in these investigations, and the problem was reduced to studying geodesic flows (Krylov N. S. Works on Foundations of Statistical Physics, Princeton Univ. Press, 1979). Ensemble averaging yields the following expression for the effective stochastization time:

$$\tau_e = s\,\frac{N^{1/3}}{c^{1/2}}\,\tau_c, \tag{1}$$

where

$$\tau_c = [L^3/(GM)]^{1/2}$$

is the crossing time. Here G is the gravitational constant, N is the number of stars, M is the mass of the system, L is the effective system scale, c is the mean square of the dimensionless acceleration y (the unit of acceleration $a^{2/3} = (4/15)^{2/3}(2\pi GM/N)$ was chosen according to Chandrasekhar S. (Rev. Mod. Phys., 1943, **15**, 1)). The dimensionless structural factor s depends on the shape of the system and the density law.

Hence, it is necessary to find c in Eq. (1) and establish the dependence $c = c(N)$. Results of previous authors contradict one another (Gurzadyan V. G. & Savvidi G. K., A&A, 1986, **160**, 203; Rastorguev A. S. & Sementsov V. N., *op. cit.*; Ovod D. V., *op. cit.*). A numerical value of c is influenced mainly by behavior of $F(y)$, the distribution function of the dimensionless random acceleration, for large values of its argument. When using the famous Holtsmark distribution (that corresponds to full neglecting correlations in stellar velocities) the integral for c diverges, and it is necessary to truncate $F(y)$ at some y_c. Results do depend on the value of y_c. Then various dependences $c = c(N)$ and various (mainly, unrealistic) expressions for τ_e/τ_c will be found. The only truncation having physical justification was proposed by Rastorguev A. S. & Sementsov V. N. (*op. cit.*), and then

$$\tau_e \propto N^{1/5}\tau_c.$$

In our previous works (Ovod D. V., Ossipkov L. P., Variable Stars, the Galactic Halo and Galaxy Formation, Sternberg Astron. Inst., 2010, p. 145; Ovod D. V. *op. cit.*) we followed Petrovskaya I. V. (Sov. Astron. Lett., 1986, **12**, 237) and put

$$F(y) = C \begin{cases} H(y), & y \le y^*, \\ P(y), & y > y^*. \end{cases}$$

Here $H(y)$ is the Holtsmark function, and

$$P(y) = B\,\frac{1}{y^3}\,\mathrm{e}^{-\frac{3}{2}\alpha^2 y^2} \tag{2}$$

is the distribution function of the dimensionless random acceleration for close encounters found by Petrovskaya I. V. (*op. cit.*). It should be emphasized that B depends on the shape of the system and does not depend on N, and $\alpha \propto N^{-2/3}$. Having approximated the Holtsmark function by the distribution of acceleration from the nearest neighbour (e. g. Ossipkov L. P. (*op. cit.*))

$$W(y) = \frac{3}{2}\,\frac{1}{y^{5/2}}\,\exp\left(-\frac{1}{y^{3/2}}\right),$$

we found y^* that made both parts of $F(y)$ meet. It is important that y^* is finite and (for large N) does not depend on N. The constant C is determined by the condition

$$\int_0^{+\infty} F(y)\mathrm{d}y = 1.$$

It is easy to find that

$$C = \left[\mathrm{e}^{-y^*} + \frac{B}{2y^{*2}}\,E_2\left(\frac{3}{2}\alpha^2 y^{*2}\right)\right]^{-1}$$

with

$$E_2(z) = \int_1^{+\infty} \frac{\mathrm{e}^{-zt}}{t^2}\mathrm{d}t.$$

One can see that C is finite when $N \to \infty$.

Then

$$c \approx C\left[\int_0^{y^*} y^2 W(y)\mathrm{d}y + \int_{y^*}^{+\infty} y^2 P(y)\mathrm{d}y\right]. \tag{3}$$

Substituting (2) to Eq. (3) we find that

$$\int_{y^*}^{+\infty} y^2 P(y)\mathrm{d}y = \frac{1}{2}\,BE_1\left(\frac{3}{2}\alpha^2 y^{*2}\right),$$

where

$$E_1(z) = \int_z^{+\infty} \frac{\mathrm{e}^{-t}}{t}\mathrm{d}t$$

is the exponential integral function. When the number of stars is large, $N \gg 1$, we see that $\alpha \ll 1$, and

$$\int_{y^*}^{+\infty} y^2 P(y)\mathrm{d}y \approx -B\ln\alpha \approx \frac{2}{3}B\ln N,$$

while the first integral in Eq. (3) remains finite. Hence,

$$c \approx \frac{2}{3}CB\ln N, \qquad N \gg 1.$$

Substituting this estimate to Eq. (1) we find that the stochastization time

$$\tau_e \propto \tau_c N^{1/3} / (\ln N)^{1/2}. \tag{4}$$

The dependence $c = c(N)$ is due to correlations. We conclude that for large N the influence of correlations is relatively weak.

The timescale of dynamical evolution corresponding to (4) is close to the one suggested by Genkin I. L. (Astron. Tsirk., 1969, No. 507, 4; Sov. Phys.–Doklady, 1971, **16**, 261) and, later, by Gurzadyan V. G. & Kocharyan A. A. (A&A, 2009, **160**, 203) who found

$$\tau_e/\tau_c \propto N^{1/3}.$$

Stochastization due to effects of stellar encounters can play significant role in evolution of globular clusters and dwarf galaxies.

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