ISSN 0236-2457

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Published by the Eurasian Astronomical Society
and Sternberg Astronomical Institute

No.1571, 2012 March 13

# О РЕЛАКСАЦИИ В ЗВЁЗДНЫХ СИСТЕМАХ

## On Relaxation in Stellar Systems

**Abstract.** Приложение идей эргодической теории к изучению динамики звёздных систем — один из возможных подходов к решению парадокса релаксации. Однако применяя разработанный в рамках такого подхода метод оценки времени релаксации к пространственно-однородной модели, мы сталкиваемся с необходимостью производить обрезание распределения Хольцмарка на некотором прицельном расстоянии, тем самым обуславливая конечный результат. В данной статье предложен способ избежать процедуры обрезания — использовать распределение Петровской. Результаты соответствующих расчётов подтверждают короткую шкалу релаксации в звёздных системах.

### 1 Introduction

According to the classical theory by Jeans and Chandrasekhar the relaxation time (the timescale for distribution of velocities in the system to turn into Maxwellian one) for nonrotating systems

$$\tau_r \propto \tau_c \frac{N}{\log N} , \tag{1}$$

where $\tau_c = (L^3/GM)^{1/2}$ is the crossing time. Here $G$ is the gravitational constant, $N$ is the number of stars, $L$ is the system scale, and $M$ is its mass.

The problem is that being estimated according to (1) the relaxation time for galaxies exceeds Hubble's time by 3–5 orders of magnitude that means that present state of the systems is fully determined by initial conditions. This seems to be most unlikely from the physical point of view. The paradox was discussed for the first time by Ogorodnikov (Dynamics of Stellar Systems, London, 1965) and since then it has been known as the main paradox of the classical stellar dynamics.

### 2 Revision of the classical theory

One of the possible ways to solve the problem is to revise the classical theory of relaxation taking into account the impact of the smoothed field while Jeans and Chandrasekhar dealt only with force due to binary star-star encounters. (For a brief overview of all approaches suggested to solve the paradox see *On the fundamental paradox of stellar dynamics* (Ossipkov L.P., A&A Transact., 2006, **25**, 123).) Gurzadyan and Savvidy (A&A, 1986, **160**, 203) tried to apply the ergodic theory to the problem. The paradigm comprises two main steps: 1) applying the Maupertuis principle it is possible to study the dynamics of a gravitating $N$-body system with a fixed value of energy $E$ considering the movement along a geodesic of a point representing the whole system in a certain $3N$-dimensional Riemannian space;

2) supposing that the smoothed gravitational potential is steady and the virial theorem is valid and following the statistical approach one can find

$$\tau_e = s\frac{N^{1/3}}{c^{1/2}}\tau_c \ , \tag{2}$$

where $\tau_e$ is the effective relaxation time, $s$ is the structural factor depending on the form of the system and the density law, $c = \int_0^\infty y^2 H(y)\,dy$ and $H(y)$ is the distribution function of the dimensionless random acceleration $y$. For homogeneous stellar systems $H(y)$ is known to be Holtsmark's distribution. Since the second moment for the distribution diverges, Gurzadyan and Savvidy decided on truncation radius to be

$$r_c = \frac{4GM}{\sigma^2} \ , \tag{3}$$

$\sigma$ being the mean velocity, and find

$$\tau_e \propto \tau_c N^{1/3}. \tag{4}$$

Rastorguev and Sementsov (Astron. Lett, 2006, **32**, 16) reconsidered the approach, introduced the term stochastisation time (i. e., effective relaxation time) and truncated Holtsmark's distribution at

$$r_{\rm RS} = (6.5{d_0}^3{p_\perp}^2)^{1/5} \ , \tag{5}$$

where $d_0 = \frac{1}{2}n^{1/3}$, $n$ being the mean number density, $p_\perp = 2Gm/\sigma^2$, $m$ being the mass of a star. As the result they come to conclusion that

$$\tau_e \propto \tau_c N^{1/5}. \tag{6}$$

Our calculations proved that the exponent of the number $N$ does depend on the truncation radius chosen for Holtsmark's function (Ovod D.V, Control Processes and Stability, 2010, **XLI**, 199).

**3 Petrovskaya's distribution**

With the truncating procedure being an obvious drawback of the approach, discrediting the results obtained, we face the necessity of using a distribution function with a finite second moment. An appropriate distribution was found by Petrovskaya (Soviet Ast. Lett., 1986, **12**, 237). Assuming binary star-star encounters to be a purely discontinuous random process, she managed to generalize Holtsmark's distribution. The new distribution function is

$$M(y) = \begin{cases} H(y), & y \le y^* \, ; \\ H_1(y), & y \ge y^* \, . \end{cases} \tag{7}$$

Here $H(y)$ is the Holtsmark's function, $H_1(y)$ is the distribution function for close encounters,

$$H_1(y) = \frac{4\sqrt{6\pi}}{Q^2}\frac{1}{y^3}\mathrm{e}^{-\frac{3}{2}\alpha^2 y^2} \ , \tag{8}$$

where $\alpha = (Q/4^{1/3})\varepsilon^{-1/3}(1+\gamma^2)N^{-2/3}$, $Q = (4/15)^{2/3}2\pi$, $\varepsilon$ is a factor characterizing the shape of the system. Having approximated Holtsmark's function by the distribution of acceleration from the nearest neighbour, we found $y^*$ which made both parts of $M(y)$ meet.

Calculations of $\tau_e/\tau_c$ with Petrovskaya's distribution function show that this ratio is proportional to $N^{0.30}$ for non-rotating systems and to $N^{0.28}$ for systems with various rotation, although the proportionality coefficients differ for systems with different characteristics. In whole this proves the short relaxation time-scale in stellar systems, stated for the first time by Genkin (Soviet Phys.– Doklady, 1971, **16**, 261). Recently Gurzadyan and Kocharyan (A&A, 2009, **505**, 629) obtained the same results as those of Genkin, which is not surprising, their assumptions being rather similar.

As for the influence of rotation on the relaxation process, we can draw the same conclusion as the one based on calculations with the truncating procedure: rotation slows down evolution in stellar systems.

In addition, it should be noticed that $\tau_e$ and $\tau_c$ are of the same order for small systems ($N \leq 10^5$) while in systems with the number of stars $N = 10^6 - 10^{12}$ relaxation slows down. A possible explanation is that in systems with more stars the impact of the smoothed field becomes more essential.

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